Quantum gravity and gravitational-wave astronomy

in collaboration with S. Kuroyanagi, S. Marsat, M. Sakellariadou, N. Tamanini, G. Tasinato

Gianluca Calcagni

Institute of Matter Structure – CSIC

January 15th, 2019

Quantum gravity (QG)

- QG is an umbrella name labeling all theories attempting to quantize the gravitational force.
- Three main aims or motivations: (1) unify the forces of Nature; (2) resolve the singularities of general relativity (big bang, black holes); (3) resolve the cosmological constant problem.

Quantum gravity (QG)

- QG is an umbrella name labeling all theories attempting to quantize the gravitational force.
- Three main aims or motivations: (1) unify the forces of Nature; (2) resolve the singularities of general relativity (big bang, black holes); (3) resolve the cosmological constant problem.

Commonplaces:

- Gravity has not been quantized consistently so far.

Quantum gravity (QG)

- QG is an umbrella name labeling all theories attempting to quantize the gravitational force.
- Three main aims or motivations: (1) unify the forces of Nature; (2) resolve the singularities of general relativity (big bang, black holes); (3) resolve the cosmological constant problem.

Commonplaces:

- Gravity has not been quantized consistently so far. It has, in several different ways.

Quantum gravity (QG)

- QG is an umbrella name labeling all theories attempting to quantize the gravitational force.
- Three main aims or motivations: (1) unify the forces of Nature; (2) resolve the singularities of general relativity (big bang, black holes); (3) resolve the cosmological constant problem.

Commonplaces:

- Gravity has not been quantized consistently so far. It has, in several different ways.
- Quantum gravities are too abstract and they do not make physical predictions.

Quantum gravity (QG)

- QG is an umbrella name labeling all theories attempting to quantize the gravitational force.
- Three main aims or motivations: (1) unify the forces of Nature; (2) resolve the singularities of general relativity (big bang, black holes); (3) resolve the cosmological constant problem.

Commonplaces:

- Gravity has not been quantized consistently so far. It has, in several different ways.
- Quantum gravities are too abstract and they do not make physical predictions. Many do.

Quantum gravity (QG)

- QG is an umbrella name labeling all theories attempting to quantize the gravitational force.
- Three main aims or motivations: (1) unify the forces of Nature; (2) resolve the singularities of general relativity (big bang, black holes); (3) resolve the cosmological constant problem.

Commonplaces:

- Gravity has not been quantized consistently so far. It has, in several different ways.
- Quantum gravities are too abstract and they do not make physical predictions. Many do.
- Quantum gravity cannot be tested at scales above Planck (SM, post-inflationary).

Quantum gravity (QG)

- QG is an umbrella name labeling all theories attempting to quantize the gravitational force.
- Three main aims or motivations: (1) unify the forces of Nature; (2) resolve the singularities of general relativity (big bang, black holes); (3) resolve the cosmological constant problem.

Commonplaces:

- Gravity has not been quantized consistently so far. It has, in several different ways.
- Quantum gravities are too abstract and they do not make physical predictions. Many do.
- Quantum gravity cannot be tested at scales above Planck (SM, post-inflationary). It can*.

Common "wisdom" in QG

In perturbative QG, effects are important at high curvature, high energy, short distances, or early times. Example:

$$\mathcal{L} = R + O(R^2)$$

Common “wisdom” in QG

In perturbative QG, effects are important at high curvature, high energy, short distances, or early times. Example:

$$\mathcal{L} = R + O(R^2)$$

Dimensional argument: in homogeneous cosmology, with only the Planck scale $\ell_{\rm Pl}$ and the Hubble scale H available, perturbative quantum corrections are of the form

$$(\ell_{\rm Pl} H)^n \stackrel{\text{today}}{=} (\ell_{\rm Pl} H_0)^n \sim (10^{-60})^n, \qquad n = 1, 2, 3, \ldots$$

Thank you for your attention!

Thank you for your attention!

. . . Hold on . . .

Motivations

How to evade the dimensional argument:

1. change topic;

Motivations

How to evade the dimensional argument:

1. change topic;
2. go at early times when H was much bigger (inflation);

Motivations

How to evade the dimensional argument:

1. change topic;
2. go at early times when H was much bigger (inflation);
3. consider models with a third scale $L \gg \ell_{\rm Pl}$ in the system, quantum corrections $\sim \ell_{\rm Pl}^a H^b L^c$ with $a - b + c = 0$, not all of them small. Example: LQC with anomaly cancellation [Bojowald, G.C., Tsujikawa, PRL 2011];

Motivations

How to evade the dimensional argument:

1. change topic;
2. go at early times when H was much bigger (inflation);
3. consider models with a third scale $L \gg \ell_{\rm Pl}$ in the system, quantum corrections $\sim \ell_{\rm Pl}^a H^b L^c$ with $a - b + c = 0$, not all of them small. Example: LQC with anomaly cancellation [Bojowald, G.C., Tsujikawa, PRL 2011];
4. consider non-perturbative QG effects.

Motivations

How to evade the dimensional argument:

1. change topic;
2. go at early times when H was much bigger (inflation);
3. consider models with a third scale $L \gg \ell_{\rm Pl}$ in the system, quantum corrections $\sim \ell_{\rm Pl}^a H^b L^c$ with $a - b + c = 0$, not all of them small. Example: LQC with anomaly cancellation [Bojowald, G.C., Tsujikawa, PRL 2011];
4. consider non-perturbative QG effects.

QG and GWs:

- Growing literature, but little or no knowledge about non-perturbative effects. Starting to fill this gap.
- Any imprint in GW production or propagation?
- Model-independent analysis?
- Which theories of QG are most promising, if any?

Universal features in quantum gravity

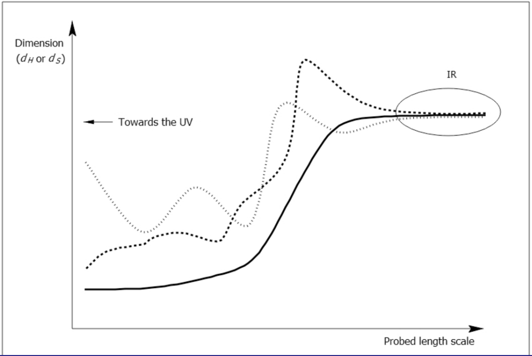
All theories of quantum gravity have something in common:

- Fuzziness: intrinsic uncertainty in measurements of times and distances.
- Dimensional flow: Changing behaviour of correlation functions, spacetime with scale-dependent dimension d. $d < 4$ in the UV. Universal feature in QG ['t Hooft 1993; Carlip 2009; G.C. PRL 2010].

Dimensional flow

Universal non-perturbative effect (present at all scales): spacetime Hausdorff and spectral dimensions change with probed scale.

$d_{\rm H}$: scaling of volumes. $d_{\rm S}$: scaling of dispersion relations.



Testing dimensional flow?

Strategy:

1. Concentrate on GW propagation (simpler than production).
2. Model-independent relation between luminosity distance d_L and GW amplitude h.
3. Use standard sirens to place model-independent constraints on this relation.
4. Apply them to specific QG theories.

Luminosity distance d_L

Flux = power per unit area

$$\mathrm{F} =: \frac{\mathrm{L}}{4\pi d_L^2}$$

Proper distance $r = \tau_0 - \tau(z)$, redshift $1 + z = a_0/a$.

$$d_L = \frac{a_0^2}{a} r = (1+z) \int_{t(z)}^{t_0} \frac{\mathrm{d}t}{a} = (1+z) \int_0^z \frac{\mathrm{d}z}{H}$$

Luminosity distance d_L

Flux = power per unit area

$$\mathrm{F} =: \frac{\mathrm{L}}{4\pi d_L^2}$$

Proper distance $r = \tau_0 - \tau(z)$, redshift $1 + z = a_0/a$.

$$d_L = \frac{a_0^2}{a} r = (1+z)\int_{t(z)}^{t_0} \frac{\mathrm{d}t}{a} = (1+z)\int_0^z \frac{\mathrm{d}z}{H}$$

Two possible modifications in QG: integration measure (distances) and/or Hubble parameter (dynamics).

GW amplitude h in D dimensions

Full calculation in the *local wave zone* $r \gg s \sim \lambda \sim \frac{1}{\omega}$ [Cardoso, Dias, Lemos 2003]: leading term of a $\omega r \gg 1$ expansion of $h(t, r)$.

GW amplitude h in D dimensions

Full calculation in the *local wave zone* $r \gg s \sim \lambda \sim \frac{1}{\omega}$ [Cardoso, Dias, Lemos 2003]: leading term of a $\omega r \gg 1$ expansion of $h(t, r)$.

Scaling argument (faster, simpler, useful)

$$S_\varphi = \int \mathrm{d}^D x \left(\frac{1}{2}\varphi\Box_\eta\varphi + \varphi J\right), \quad [\varphi] = \frac{D-2}{2} \Rightarrow \varphi \simeq \frac{\mathcal{F}(t,r)}{r^{\frac{D-2}{2}}} \overset{D=4}{\propto} \frac{1}{r}.$$

GW amplitude h in D dimensions

Full calculation in the *local wave zone* $r \gg s \sim \lambda \sim \frac{1}{\omega}$ [Cardoso, Dias, Lemos 2003]: leading term of a $\omega r \gg 1$ expansion of $h(t, r)$.

Scaling argument (faster, simpler, useful)

$$S_\varphi = \int \mathrm{d}^D x \left(\frac{1}{2}\varphi\Box_\eta\varphi + \varphi J\right), \quad [\varphi] = \frac{D-2}{2} \Rightarrow \varphi \simeq \frac{\mathcal{F}(t,r)}{r^{\frac{D-2}{2}}} \overset{D=4}{\propto} \frac{1}{r}.$$

Cosmological propagation (flat FLRW) of the scalar $\varphi := h_{+,\times}/\kappa$

$$\Box\varphi = -\frac{1}{a^2}[\partial_\tau^2 + (D-2)\mathcal{H}\partial_\tau - \nabla^2]\varphi = -J\,, \qquad \mathcal{H} := \frac{a'}{a}\,.$$

Rescaling $w := a^{\frac{D-2}{2}}\varphi \Rightarrow (\partial_\tau^2 - a''/a - \nabla^2)w = a^{\frac{D+2}{2}}J$.

GW amplitude h in D dimensions

Full calculation in the *local wave zone* $r \gg s \sim \lambda \sim \frac{1}{\omega}$ [Cardoso, Dias, Lemos 2003]: leading term of a $\omega r \gg 1$ expansion of $h(t,r)$.

Scaling argument (faster, simpler, useful)

$$S_\varphi = \int \mathrm{d}^D x \left(\frac{1}{2}\varphi\Box_\eta\varphi + \varphi J\right), \quad [\varphi] = \frac{D-2}{2} \Rightarrow \varphi \simeq \frac{\mathcal{F}(t,r)}{r^{\frac{D-2}{2}}} \overset{D=4}{\propto} \frac{1}{r}.$$

Cosmological propagation (flat FLRW) of the scalar $\varphi := h_{+,\times}/\kappa$

$$\Box\varphi = -\frac{1}{a^2}[\partial_\tau^2 + (D-2)\mathcal{H}\partial_\tau - \nabla^2]\varphi = -J\,, \qquad \mathcal{H} := \frac{a'}{a}\,.$$

Rescaling $w := a^{\frac{D-2}{2}}\varphi \Rightarrow (\partial_\tau^2 - a''/a - \nabla^2)w = a^{\frac{D+2}{2}}J$. Drop a''/a for sub-horizon modes, get $\varphi \sim (ar)^{-\frac{D-2}{2}}$. $r \to ar$ rule:

$$h \simeq \frac{\kappa\mathcal{F}(t,r)}{d_L^{\frac{D-2}{2}}} \overset{D=4}{\propto} \frac{1}{d_L}$$

Effective dynamics in QG

Assumptions: continuum limit, effective action

$$S=\frac{1}{2}\int \mathrm{d}\varrho(x)\,\varphi\mathcal{K}\varphi,\qquad [\varrho]=-d_{\rm H},\qquad [\varphi]=\frac{d_{\rm H}-[\mathcal{K}]}{2}$$

Effective dynamics in QG

Assumptions: continuum limit, effective action

$$S = \frac{1}{2}\int \mathrm{d}\varrho(x)\, \varphi\mathcal{K}\varphi, \qquad [\varrho] = -d_{\mathrm{H}}, \qquad [\varphi] = \frac{d_{\mathrm{H}} - [\mathcal{K}]}{2}$$

Modified dispersion relation, e.g., $\mathcal{K}(-k^2) = -\ell_*^{2-2\beta}k^2 + k^{2\beta}$.

Effective dynamics in QG

Assumptions: continuum limit, effective action

$$S=\frac{1}{2}\int \mathrm{d}\varrho(x)\,\varphi\mathcal{K}\varphi,\quad [\varrho]=-d_{\rm H},\quad [\varphi]=\frac{d_{\rm H}-[\mathcal{K}]}{2}$$

Modified dispersion relation, e.g., $\mathcal{K}(-k^2)=-\ell_*^{2-2\beta}k^2+k^{2\beta}$.
Return probability and spectral dimension

$$\mathcal{P}(\sigma)\propto\int \mathrm{d}\tilde{\varrho}(k)\,\mathrm{e}^{-\sigma\ell_*^{[\mathcal{K}]}\mathcal{K}(-k^2)},\quad d_{\rm S}:=-2\frac{\mathrm{d}\ln\mathcal{P}(\sigma)}{\mathrm{d}\ln\sigma}=2\frac{d_{\rm H}^k}{[\mathcal{K}]}$$

Effective dynamics in QG

Assumptions: continuum limit, effective action

$$S=\frac{1}{2}\int \mathrm{d}\varrho(x)\,\varphi\mathcal{K}\varphi,\qquad [\varrho]=-d_{\rm H},\qquad [\varphi]=\frac{d_{\rm H}-[\mathcal{K}]}{2}$$

Modified dispersion relation, e.g., $\mathcal{K}(-k^2)=-\ell_*^{2-2\beta}k^2+k^{2\beta}$.
Return probability and spectral dimension

$$\mathcal{P}(\sigma)\propto\int \mathrm{d}\tilde{\varrho}(k)\,\mathrm{e}^{-\sigma\ell_*^{[\mathcal{K}]}\mathcal{K}(-k^2)},\qquad d_{\rm S}:=-2\frac{\mathrm{d}\ln\mathcal{P}(\sigma)}{\mathrm{d}\ln\sigma}=2\frac{d_{\rm H}^k}{[\mathcal{K}]}$$

Anomalous scaling

$$\boxed{\Gamma:=\frac{d_{\rm H}}{2}-\frac{d_{\rm H}^k}{d_{\rm S}}\,,\qquad d_{\rm S}\neq 0}$$

UV geometry in QG

	$d_{\rm H}^{\rm UV}$	$d_{\rm H}^{k,\rm UV}$	$d_{\rm S}^{\rm UV}$	γ	$\Gamma_{\rm meso} \gtrsim 1$
GFT/spin foams/LQG	2	4	$1 \leq d_{\rm S}^{\rm UV} < 4$	$-3 \leq \gamma < 0$	✓
Causal dynamical triangulations (phase C)	4	4	$3/2$	$-2/3$	
κ-Minkowski bicovariant ∇^2 (c.i.m.)	1	3	3	$-1/2$	
κ-Minkowski bicross-product ∇^2 (c.i.m.)	1	3	6	0	
Stelle gravit	4	4	2	0	
String theory (low-energy limit)	D	D	2	0	
Asymptotic safety	4	4	2	0	
Hořava–Lifshitz gravity	4	4	2	0	
κ-Minkowski relative-locality ∇^2 (c.i.m.)	1	3	$+\infty$	$1/2$	
κ-Minkowski bicovariant ∇^2 (o.m.)	4	3	3	1	
κ-Minkowski bicross-product ∇^2 (o.m.)	4	3	6	$3/2$	✓
κ-Minkowski relative-locality ∇^2 (o.m.)	4	3	$+\infty$	2	✓
Padmanabhan's non-local model	4	4	$+\infty$	2	✓

GW amplitude in QG

$$h \propto \frac{1}{d_L^\Gamma} \qquad \Rightarrow \qquad h \overset{\rm UV}{\sim} \frac{1}{d_L^\gamma}\,, \qquad h \overset{\rm IR}{\sim} \frac{1}{d_L^{\frac{D-2}{2}}}\,.$$

GW amplitude in QG

$$h \propto \frac{1}{d_L^\Gamma} \qquad \Rightarrow \qquad h \overset{\text{UV}}{\sim} \frac{1}{d_L^\gamma}\,, \qquad h \overset{\text{IR}}{\sim} \frac{1}{d_L^{\frac{D-2}{2}}}\,.$$

$$\boxed{h \propto \frac{1}{d_L^{\rm GW}}\,, \qquad d_L^{\rm GW} = d_L^{\rm EM}\left[1 \pm |\gamma-1|\left(\frac{d_L^{\rm EM}}{\ell_*}\right)^{\gamma-1}\right]}$$

GW amplitude in QG

$$h \propto \frac{1}{d_L^\Gamma} \qquad \Rightarrow \qquad h \overset{\rm UV}{\sim} \frac{1}{d_L^\gamma}\,, \qquad h \overset{\rm IR}{\sim} \frac{1}{d_L^{\frac{D-2}{2}}}\,.$$

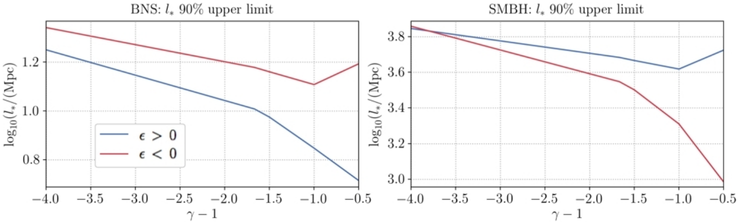
$$\boxed{h \propto \frac{1}{d_L^{\rm GW}}\,, \qquad d_L^{\rm GW} = d_L^{\rm EM}\left[1 \pm |\gamma - 1|\left(\frac{d_L^{\rm EM}}{\ell_*}\right)^{\gamma-1}\right]}$$

Very similar to models with extra dimensions [Deffayet, Menou 2007; Pardo et al. 2018; Abbott et al. 2018]

$$h \propto \frac{1}{d_L^{\rm GW}}\,, \qquad d_L^{\rm GW} = d_L^{\rm EM}\left[1 + \left(\frac{d_L^{\rm EM}}{R_{\rm c}}\right)^{n_{\rm c}}\right]^{\frac{D-4}{2n_{\rm c}}}$$

Constraining QG with standard sirens

Fix γ, infer ℓ_*. Sirens: real BNS GW170817 (LIGO-Virgo) and simulated SMBH (LISA).



UV regime of QG avoids constraints, GW propagation little affected.

Constraining QG with standard sirens. 2

GW interferometers: $\Delta d_L/d_L \sim 0.01 - 0.2$. Detectable QG effect if $\gamma \gtrsim 1$, even when $\ell_* = O(\ell_{\rm Pl})$:

$$\left(\frac{d_L^{\rm EM}}{\ell_*}\right)^{\gamma-1} \gtrsim 10^{-3} - 10^{-1}$$

Constraining QG with standard sirens. 2

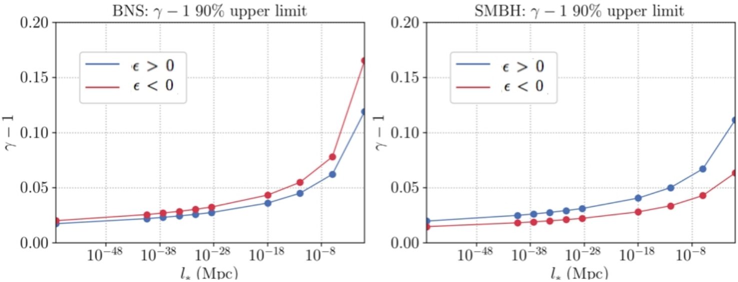
GW interferometers: $\Delta d_L/d_L \sim 0.01 - 0.2$. Detectable QG effect if $\gamma \gtrsim 1$, even when $\ell_* = O(\ell_{\rm Pl})$:

$$\left(\frac{d_L^{\rm EM}}{\ell_*}\right)^{\gamma-1} \gtrsim 10^{-3} - 10^{-1}$$

Concrete theories such that $\gamma \gtrsim 1$:

- Non-commutative κ-Minkowski spacetime
  $\Gamma_{\rm meso} \simeq 1 + \frac{5}{96\pi}\frac{\ell_{\rm Pl}^2}{\ell^2} \sim 1 + 10^{-120}$.
- Effective non-local QFT near BH horizon
  $\Gamma_{\rm meso} \simeq 1 + \frac{5\pi}{2}\frac{\ell_{\rm Pl}^2}{\ell^2} \sim 1 + 10^{-120}$.
- Quantum gravities with discrete pre-geometries: GFT, spin foams, LQG. $\Gamma_{\rm meso}$ strongly state dependent.

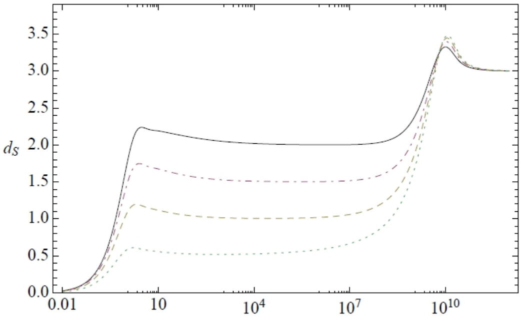
Fixed ℓ_*, infer γ



$$\boxed{\Gamma_{\rm meso} - 1 < 0.02, \qquad \frac{d_{\rm S}^{\rm meso} - 4}{4} < 0.08}$$

Future work: testing GFT/spin foams/LQG

Numerical analysis of dimensional flow [G.C., Oriti, Thürigen 2013,2014,2015]



$\Gamma_{\rm meso}$ can be calculated from realistic quantum states of geometry.

Excluding some LQC semi-classical states

Effective-dynamics LQC. Quantum corrections $\delta_{\rm Pl}$ can be large [Bojowald, G.C., Tsujikawa, 2011a,b].

Excluding some LQC semi-classical states

Effective-dynamics LQC. Quantum corrections $\delta_{\rm Pl}$ can be large [Bojowald, G.C., Tsujikawa, 2011a,b].

Mukhanov equation ($w_k := a_{\rm LQC} h_k$):

$$w_k'' + \left[(1+2\alpha_0\delta_{\rm Pl})k^2 - \frac{a_{\rm LQC}''}{a_{\rm LQC}}\right] w_k = 0\,, \qquad a_{\rm LQC} := a\left(1-\frac{\alpha_0}{2}\delta_{\rm Pl}\right)$$

$$\frac{d_L^{\rm GW}(z)}{d_L^{\rm EM}(z)} = \frac{a}{a_{\rm LQC}} \simeq 1 + \frac{1}{2}\alpha_0\delta_{\rm Pl}$$

Excluding some LQC semi-classical states

Effective-dynamics LQC. Quantum corrections $\delta_{\rm Pl}$ can be large [Bojowald, G.C., Tsujikawa, 2011a,b].

Mukhanov equation ($w_k := a_{\rm LQC} h_k$):

$$w_k'' + \left[(1+2\alpha_0\delta_{\rm Pl})k^2 - \frac{a_{\rm LQC}''}{a_{\rm LQC}}\right] w_k = 0\,, \qquad a_{\rm LQC} := a\left(1 - \frac{\alpha_0}{2}\delta_{\rm Pl}\right)$$

$$\frac{d_L^{\rm GW}(z)}{d_L^{\rm EM}(z)} = \frac{a}{a_{\rm LQC}} \simeq 1 + \frac{1}{2}\alpha_0\delta_{\rm Pl}$$

But the correction is constrained to be too small from the GW170817 + GRB170817A bound on propagation speed: $\alpha_0\delta_{\rm Pl} < 10^{-15}$.

Parametrizing d_L in QG

Belgacem et al. (2018) parametrization

$$\frac{d_L^{\mathrm{GW}}(z)}{d_L^{\mathrm{EM}}(z)} = \Xi_0 + \frac{1-\Xi_0}{(1+z)^n} \overset{z\ll 1}{=} 1-(1-\Xi_0)n\,z + O(z^2)$$

does not hold in QG.

Parametrizing d_L in QG

Belgacem et al. (2018) parametrization

$$\frac{d_L^{\rm GW}(z)}{d_L^{\rm EM}(z)} = \Xi_0 + \frac{1-\Xi_0}{(1+z)^n} \overset{z\ll 1}{=} 1-(1-\Xi_0)n\,z + O(z^2)$$

does not hold in QG. Alternative:

$$\frac{d_L^{\rm GW}(z)}{d_L^{\rm EM}(z)} = \Xi_0 + (1-\Xi_0)\,\mathrm{e}^{-z^m} = 1-(1-\Xi_0)\,z^m + O(z^{2m})$$

In QG, $m=\gamma-1$. Applications in modified gravity?

Strain noise and QG

Uncertainty/fuzziness at Planck scales $\ell_* = \ell_{\rm Pl}$ (intrinsic QG noise) no greater than the strain noise:

	$\mathcal{S}$ ($\mathrm{Hz}^{-1/2}$)	f (Hz)	α
LIGO/Virgo/KAGRA	10^{-23}	10^2	< 0.47
LISA	10^{-20}	10^{-2}	< 0.54
DECIGO	10^{-23}	10^{-1}	< 0.47

Model-independent bound on the small-scale Hausdorff dimension of spacetime:

$$d_{\rm H}^{\rm UV} < 1.9\,.$$

Perspective

- Different theories of QG have common non-perturbative effects present at all scales. Example: dimensional flow.
- In most theories, dimensional flow is not observable in standard-sirens observations.
- Proof of concept: model-independent rigorous bounds are possible and there is more to them than back-of-the-envelope arguments about quantum corrections.
- Future work:
  - study LQC in hybrid quantization (avoids speed bound), analytic and numerical;
  - study full GFT/spin foams/LQG states of quantum geometry, numerical;
  - study production of GW in QG (challenging).

ご清聴ありがとうございました

Muchas gracias

Kiitos paljon

Thank you

Grazie

Muito obrigado

Danke schön

Спасибо